Mapping Vegetation Properties and Flow Patterns in STAs using Wave Tests

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Why is the study of vegetation resistance important?

- Timing and attenuation of flood peaks in hydrologic systems and models depends on vegetation resistance.
- Operation of STAs require a knowledge of hydraulic transients in vegetated wetlands.
- Designing efficient treatment wetlands is primarily a hydraulic problem because of the influence of turbulence, dead flow zones, mixing and retention.
Challenges

- There is no vegetation measurements available (diameter, spacing, density, biomass)
- Access is challenging.
- Measurements of depth, slope, and flow velocity, etc. are not easy
Progress in establishing the “Science”

- ASCE J. Hydraul.
- AGU/WRR Pub.
- Workshop by Prof Heidi Nepf, MIT
- Contacts with Kadlec, R. H.
- 3-4 presentations at conferences
Recent Developments
Understanding of the Mechanics has changed

Figure 7.2: Vertical velocity profiles in open channels and vegetated wetlands
Parameterization is improving

The commonly used equation for depth-averaged force balance is

\[ g s_f = \frac{1}{2} c_D a U^2 + \frac{\tau_0}{\rho H} \]

where

\[ a = \frac{\text{frontal area}}{\text{volume}} \]

and \( a h = \text{frontal area index} \).

a used to define vegetation density
**Figure-2.** Plan view of the mesh for the vegetation cover ($V_{C_d}$) of the deep flow region and the vegetation cover ($V_{C_s}$) of the shallow flow part for the IS configurations.
In the absence of data, we used wave propagation methods, monitored wave velocity and attenuation.
Basic mathematical methods used for the formulation

- Differential calculus - Hilderbrand
- Complex variables -
- Spectral analysis –
- Linear stability theory -
- Perturbation theory – Fluid mechanics, Kundu
- Transport and dispersion of solutes - Fisher
Use of depth averaged flow

2.1. Depth-Averaged Flow Equations
St. Venant’s equations are used to analyze the shallow water waves generated in the wetlands. The St. Venant’s equations consist of a continuity equation and a momentum equation.

\[
\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0
\]  

\[
\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{q^2}{h} \right) + gh (s_f + \frac{\partial h}{\partial x} - s_0) = 0
\]

where \( h \) = water depth; \( q \) = discharge per unit width; \( g \) = gravitational acceleration; \( s_0 = - \frac{\partial z}{\partial x} \) = bed slope; \( z \) = bottom elevation; \( H = h + z \) = water level; \( s_f \) = friction slope. Figure 1 shows a definition sketch drawn
Energy Slope $S_f$ related to discharge with a smooth function.

$$\Delta q = a \Delta h + K \Delta s_f$$

$$a(h, s_f) = \frac{\partial q(h, s_f)}{\partial h}, \quad \text{and} \quad K(h, s_f) = \frac{\partial q(h, s_f)}{\partial s_f}$$

$a =$ kinematic celerity [Chow, 1956]; 
$K =$ hydraulic diffusivity, or transmissivity.
Kinematic vs Porous Media Flow

(a) Figure showing a large change in discharge with depth. Change in discharge with slope is small.

(b) Figure showing large change in discharge with slope. Change in discharge with depth is small.

large $a h$
small $K s_f$
large $\Psi$
small $k_1, k_2$

Kinematic

Diffusive
For 2-D Wave Propagation in a Shallow Water Medium

Linearization of (3.1) leads to

\[
\frac{\partial h}{\partial t} + a_x \frac{\partial h}{\partial x} + a_y \frac{\partial h}{\partial y} = K_x \frac{\partial^2 h}{\partial x^2} + K_y \frac{\partial^2 h}{\partial y^2} + 2K_{xy} \frac{\partial^2 h}{\partial x \partial y}
\]  

(3.5)

where

\[
a_x = \frac{\partial q_x}{\partial h} = \frac{\partial T}{\partial h} s_{fx}
\]  

(3.6)

\[
a_y = \frac{\partial q_y}{\partial h} = \frac{\partial T}{\partial h} s_{fy}
\]  

(3.7)

\[
K_{xx} = \frac{\partial q_x}{\partial s_{fx}} = \frac{\partial T}{\partial s_{fn}} s_{fx}^2 + T
\]  

(3.8)

\[
K_{yy} = \frac{\partial q_y}{\partial s_{fy}} = \frac{\partial T}{\partial s_{fn}} s_{fy}^2 + T
\]  

(3.9)

\[
K_{xy} = \frac{\partial q_x}{\partial s_{fy}} = \frac{\partial T}{\partial s_{fn}} s_{fy} s_{fx}
\]  

(3.10)
Choose Power law equations – For Easy Mathematics

- Discharge is a function of water depth and slope:

\[ q = f(\text{depth}, \text{slope}) = f(d, s) \]

\[ q = \frac{1}{n_b} h^{1+\gamma} s^\alpha \]
Three physical parameters to match three physical characterizations of hydraulics

- **\( \gamma \)** - Gamma – gives depth variability
- **\( \alpha \)** - Alpha – gives level of turbulence
- **\( n_b \)** - Manning’s constant characterizes the resistance

\[
q = \frac{1}{n_b} h^{1+\gamma} |s_f|^{\alpha} \text{sgn}(s_f)
\]
Field Test

STA3/4 Cell 2A Wave 1
Discharge 750 cfs,
Period 64 hour
STA3/4 Cell 2A

Waves generated using canal flow

Array of data loggers

Figure 2.1: Location of the data loggers and the IDs.

Figure 2.2: Locations of data loggers and the serial numbers. The loggers 0499, 3962 and 2835 are south of 0508 along levee.
Decay rates and wave numbers, 750 cfs

**Fig. 6.** Decay coefficients $k_1$ for STA 3/4 Cell 2A wave test with $Q = 21.2$ m$^2$/s as vectors and contours

**Fig. 7.** Wave numbers $k_2$ for STA-3/4 Cell 2A wave test with $Q = 121.2$ m$^2$/s as vectors and contours
Transmissivity

Fig. 9. Contours of transmissivity $K$ in (m$^2$/s) for Cell 2A wave test with $Q = 21.2$ m$^2$/s
Fig. 12. Contours of $1 + \gamma$ for Cell 2A wave test with $Q = 21.2 \text{ m}^2/\text{s}$; values much larger than 1 indicate possible short-circuiting
\[ \Psi = \frac{\text{Discharge through the kinematic mechanism}}{\text{Discharge through the diffusion mechanism}} \]

\[ \Psi = \frac{a(h)h}{K(h)s_f} \]

**Fig. 14.** Contours of \( \Psi \) for Cell 2A wave test with \( Q = 21.2 \text{ m}^2/\text{s} \)
Function $q(h,s_f)$ on log-log axes

Figure 7. Contours of average discharge per unit width $q$ (m$^2$/s) obtained using power law equations. The plots are made on log-log axes.
K, transmissivity regimes

- $K < 20 \text{ m}^2/\text{s}$ – dense cattail – excellent
- $20 < K < 60 \text{ m}^2/\text{s}$ – cattail with open spaces
- $60 < K \text{ m}^2/\text{s}$ – watch for short circuiting ($k > 100 \text{ m}/\text{y}$)
- $1000 < K$ – shallow overland flow
- $4000 < K$ – deep hole
- $0.001-0.01 \text{ m/s}$ hyd cond - sand
Velocity nonuniformity \((1 + \gamma)\)

- \((1 + \gamma) = 1\) uniformly distributed over depth
- Between 1 and 3 - normal
- Over 3 – Velocity non-uniformity
- 1.67 – Overland flow
Summary

- Maps for wave decay, wave speed, and resistance.
- In-situ bulk resistance functions, were graphical plots of $q(slope, depth)$, and power-law equations.
- Dimensionless numbers to detect kinematic and diffusive flow conditions or laminar/turbulent conditions in STAs.