Storm Wave Propagation Along C-111 Canal During Hurricane Irma

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Abstract

Hurricane Irma struck Florida on September 10, 2017, causing storm surges that traveled unusually long distances along Florida canals, including the C111 canal. The speed and height of the surge observed during this event are crucial for flood risk assessments. The notably large seepage component in Florida significantly impacted both the speed and amplitude of these storm surges, highlighting the need to re-evaluate standard analysis practices.

To evaluate the wave propagation, we employ analytical methods based on full St. Venant equations for canal flow, fully coupled with governing equations for groundwater flow. Spectral solutions to this problem reveal three dimensionless parameters:

(1) the ratio of inertial and gravitational forces, known as the Froude number;
(2) the ratio of frictional to gravitational forces; and
(3) the ratio of aquifer to canal storage.

We aim to track the propagation of normal tides along the C111 canal and compare it with the propagation observed during Hurricane Irma, specifically noting when the water moved inland.

By using the dimensionless numbers and analytical solutions, we will identify which forces dominate the flow behavior during hurricanes, and how variations in hurricane conditions or ground conditions influence this behavior.

Our goal is also to demonstrate the benefits of using analytical models when benchmarking numerical models.

Why is it important to know about waves

1. Tidal and hurricane surge waves travel long distances along coastal canals
2. Some large waves can cause serious flooding
2. Waves characteristics depend on canal and aquifer parameters and the wave period
3. The two key wave characteristics are (a) wave speeds and (b) wave attenuation

. Wave speeds and decay rates can be utilized to estimate the parameters of physical systems.

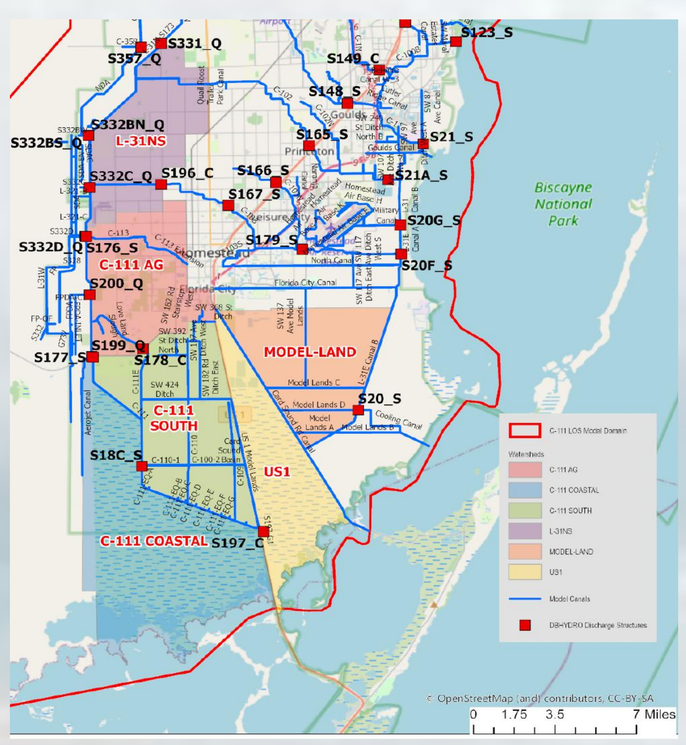
. The parameters of physical systems can, in turn, be used to estimate wave speeds and decay.

. This analysis is useful for identifying the dominant physical processes within the system.

. It is essential to ensure that the numerical models meet these benchmarks

C-111 Canal in South Florida and the S18C-S177 stretch

Location map



S18C-S177 canal properties

Observation	Current
Distance	9571 m
Width B	30.5 m
Depth d	4.57 m
Average discharge	25.5 m^3/s
Ampl. in H S18C (obs)	0.040 m
Ampl. in H S177 (obs)	0.026 m
Ratio of H ampl.	0.65
Ampl. in Q S18C (obs)	6.16 m^3/s
Ampl. in Q S177 (obs)	2.84 m^3/s
Ratio of Q ampl.	0.46
Wave speed	3.7 m/s
Parameter	
Period	12 hrs
Transmissivity	1.5 m^2/s
Manning's	0.032

Equations governing integrated canal-aquifer systems

St Venant equations for depth averaged shallow water flow

$$\frac{\partial h}{\partial t}+\frac{\partial q}{\partial x}+q_l=0$$

$$\frac{\partial q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{q^2}{h}\right)+gh\left(s_f+\frac{\partial h}{\partial x}-s_0\right)=0$$

$h(x)$ = water depth;
x = distance along the canal;
s_f = friction slope;
s_0 = bottom slope;
q_l = flow from the 1-D system to the 2-D system

$$q_l=\frac{2T_m}{B}\left(\frac{\partial H}{\partial y}\right)_{y=\delta-}\approx k_m\frac{p}{B}\frac{\Delta H}{\delta}$$

$$s_c\frac{\partial H}{\partial t}=\frac{\partial}{\partial x}\left(T_g\frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial y}\left(T_g\frac{\partial H}{\partial y}\right) \quad (1)$$

subjected to suitable initial and boundary conditions. In the equation x, y = distances along horizontal x- and y-axes; t = time; H = water head; T_g = transmissivity of the aquifer; and s_c = storage coefficient. For unconfined flow, $T_g \approx k_g h$ where k_g = hydraulic conductivity and h = aquifer thickness.

Equation governing the spectral behavior of the integrated system

$$\left(\underbrace{1}_{\frac{dh}{dt}}-\underbrace{Q_l'(\chi,\sigma)\mathbf{I}}_{seepage}\right)\left[\underbrace{1}_{\frac{dq}{dt}}\pm\underbrace{2F\hat{\lambda}\mathbf{I}}_{convective}\mp\underbrace{n\frac{1}{FP_d}\mathbf{I}}_{friction}\right]+\hat{\lambda}\left[\underbrace{\hat{\lambda}}_{gravity}\pm\underbrace{(m+n)\frac{1}{P_d}}_{friction}-\underbrace{\hat{\lambda}F^2}_{conv.accel.}\right]=0$$

$\hat{\lambda}=\lambda\Lambda$ = dimensionless wave number and decay

$\Lambda=\frac{\sqrt{gh_0}}{f_2}$ = length scale

$Q_l'(\chi,\sigma)=\frac{\bar{q}_i}{f_2}$ = influence of leakage

Analytical solutions for wave speed and decay

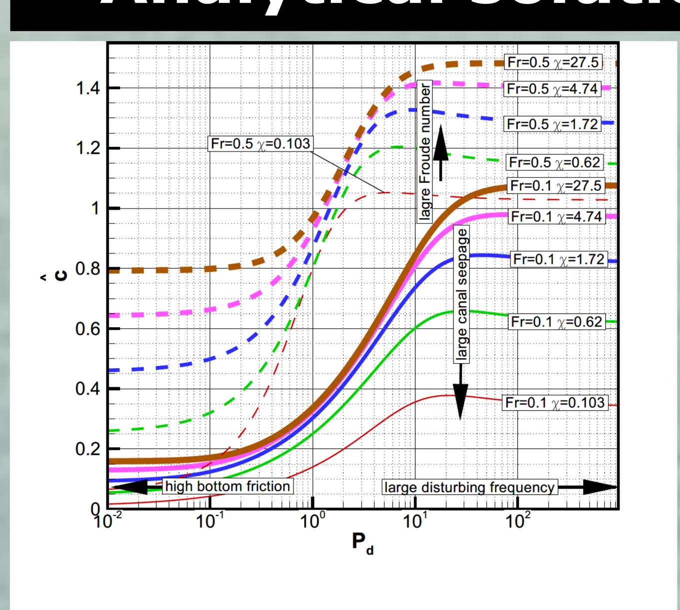


FIG. 15. Variation of δ wpth P_d plotted for F_r = 0.1, 0.5 and χ = 27.5, 4.74, 1.72 and 0.103

$$\delta \approx (1/n)ln(x_1/x_{n+1})$$

Reduced amplitude	χ
0.05	0.103 (open)
0.25	0.62
0.5	1.72
0.75	4.74
0.95	27.5 (cutoff)

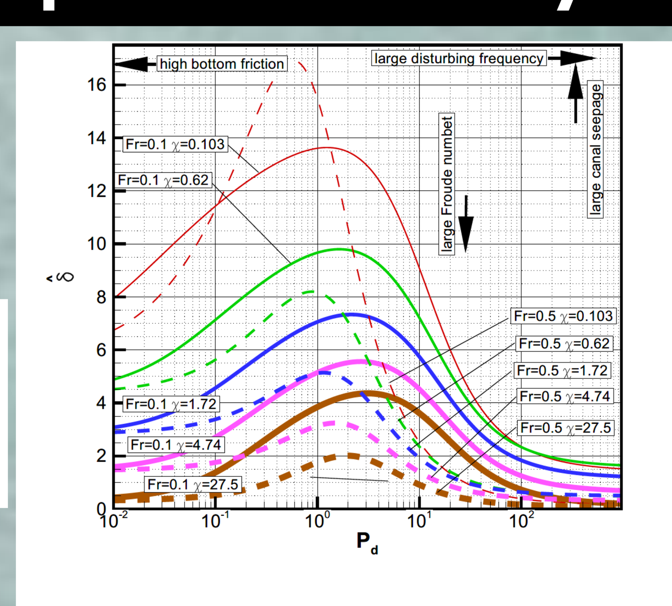
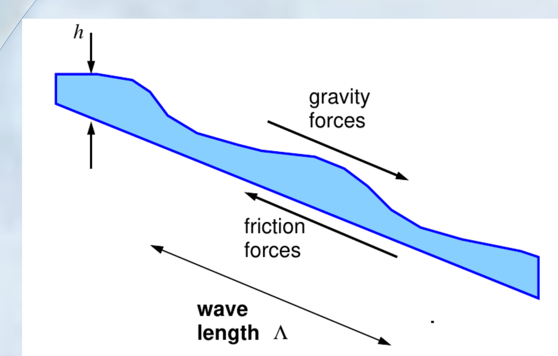


FIG. 16. Variation of $\hat{\delta}$ wpth P_d plotted for F_r = 0.1, 0.5 and χ = 27.5, 4.74, 1.72 and 0.103

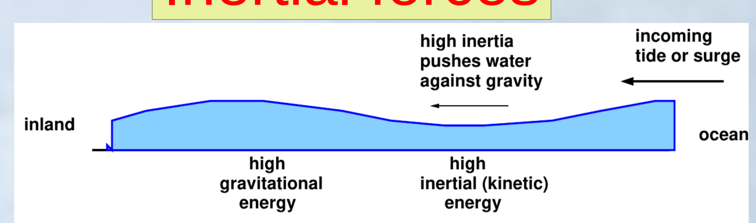
Wave Propagation Depend On

Four physical factors or three dimensionless variables

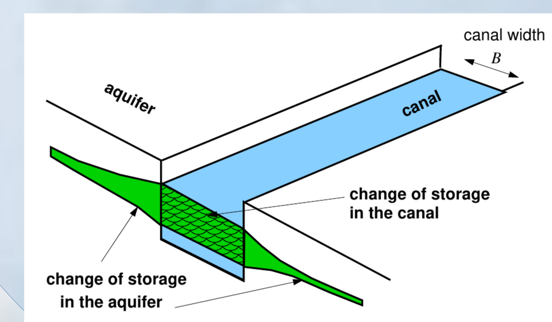
Friction forces



Inertial forces



Bank storage effects



Bank storage ratio χ

$$\chi=B\sqrt{\frac{2\pi}{PTs_c}}=\frac{\text{volume stored in canal}}{\text{volume stored in bank}}$$

B = canal width
P = wave period (or time scale)
T = aquifer transmissivity
s_c = storage coefficient

χ < 0.05 canal-aquifer fully connected
χ > 27.5 canal-aquifer disconnected

Froude number F

Froude number, F

$$F=\frac{u}{\sqrt{gh}}=\sqrt{\frac{\text{inertial forces}}{\text{gravity forces}}}$$

u = average flow velocity, Q/A
h = water depth
g = gravitational acceleration

F > 1 hydraulic jump present

Gravity friction ratio P_d

Gravity - friction ratio, P_d

$$P_d==\frac{h}{s_f\Lambda}=\frac{2\pi}{s_f P}\sqrt{\frac{h}{g}}=\frac{\text{gravitational forces}}{\text{friction forces}}$$

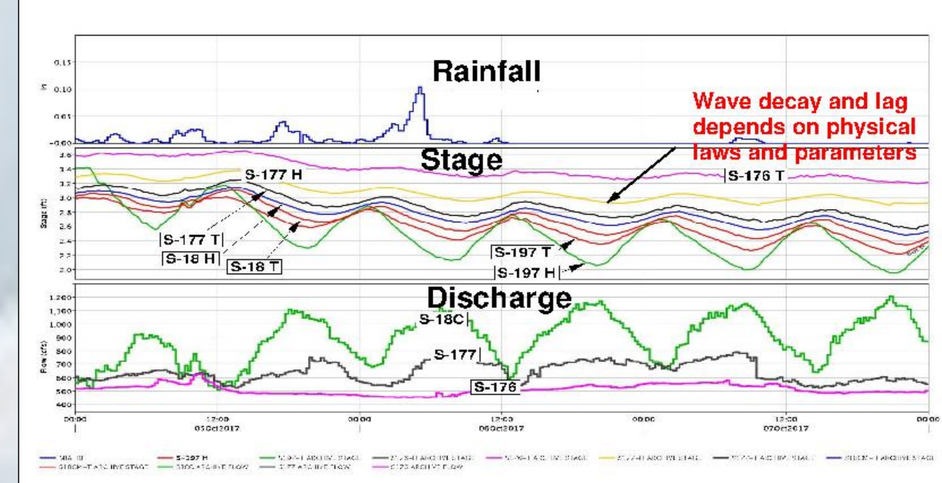
h = water depth
s_f = friction slope
g = gravitational acceleration
Λ = wave length
P = wave period

Acording to Ponce, et al. (1978), flow is inertia free if

$P_d < 1/30$

P_d < 1/30 inertia term is negligible

Normal tides



Gravity - friction ratio, P_d

$$P_d==\frac{h}{s_f\Lambda}=\frac{2\pi}{s_f P}\sqrt{\frac{h}{g}}=\frac{\text{gravitational forces}}{\text{friction forces}}$$

h = water depth
s_f = friction slope
g = gravitational acceleration
Λ = wave length
P = wave period

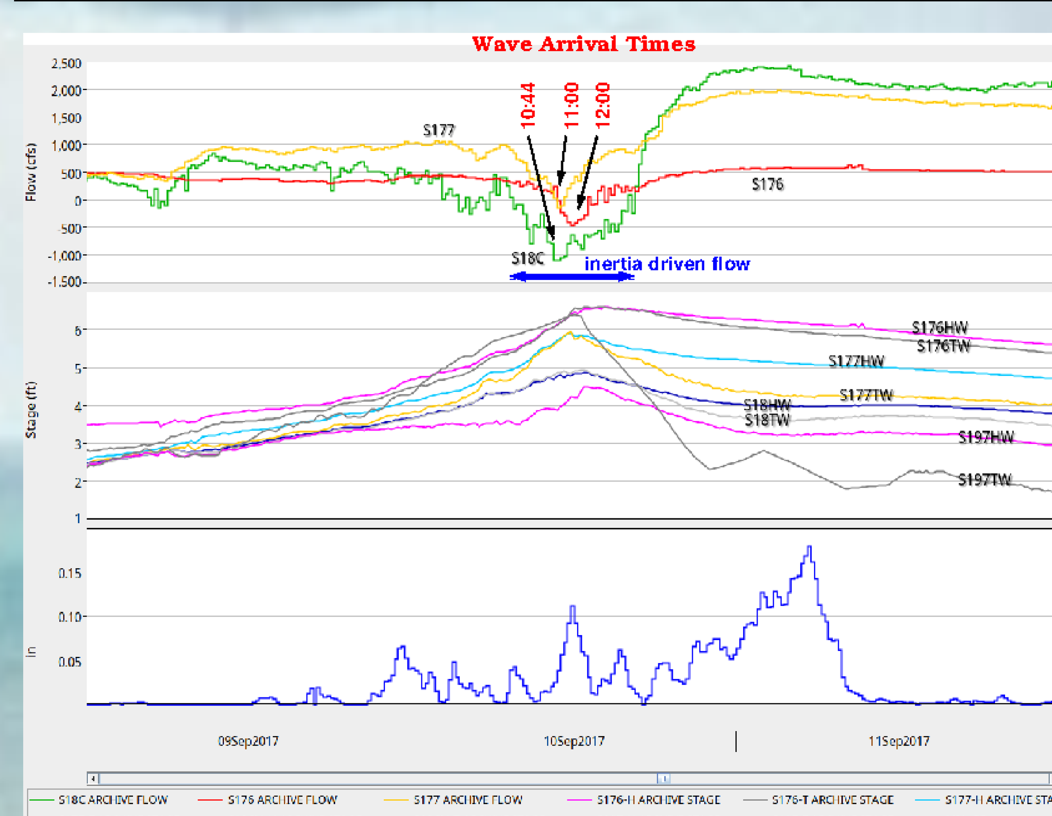
Acording to Ponce, et al. (1978), flow is inertia free if

$P_d < 1/30$

Estimated	Current
Flow velocity	0.183 m/s
Wave speed up	3.01 m/s
Wave speed dn	3.26 m/s
Froude no.	0.027
Stream interaction χ	0.671
P_d	21.9
λ_1 going up m^{-1}	6.63 $\times10^{-5}$
λ_1 going down m^{-1}	6.05 $\times10^{-5}$
Ampl in H	0.028 m
Lag in stage	5.3 hrs
Ampl. in Q decay ratio (up)	0.53
Ampl. in Q decay ratio (dn)	0.56

- Shallow water wave speed of √(gh)= 6.7 m/s slows down to 3.7 m/s due to bank storage effects
- Water level amplitude at S18C decays to 65% at S177 primarily due to bank storage effects
- Bank storage ratio χ=0.67 < 27.5 showing significance of bank storage
- P_d = 21.9 is much larger than 1/30 showing the dominance of inertia

Hurricane Irma and the forerunners



Estimated	Current	FR2	FR1
Av. flow velocity	0.179 m/s	0.179 m/s	0.183 m/s
Wave speed up	1.03 m/s	4.12 m/s	3.17 m/s
Wave speed dn	1.09 m/s	3.82 m/s	2.94 m/s
Froude no.	0.025	0.025	0.025
Stream interaction χ	0.236	0.852	0.602
P_d	3.16	41.2	20.1
λ_1 going up m^{-1}	3.41 $\times10^{-5}$	6.18 $\times10^{-5}$	5.34 $\times10^{-5}$
λ_1 going down m^{-1}	3.09 $\times10^{-5}$	5.67 $\times10^{-5}$	4.91 $\times10^{-5}$
Ampl in stage	0.40 m	0.069 m	0.092 m
Lag in stage	45 hrs	3.65 hrs	7.07 hrs
Ampl. decay ratio (up)	0.74	0.58	0.62
Ampl. decay ratio (down)	0.72	0.55	0.59

- Shallow water wave speed of √(gh)= 6.8 m/s slows down to 1.0 m/s due to bank storage effect
- Water level amplitude at S18C decays to 74% at S177 primarily due to bank storage effects
- Bank storage ratio χ=0.24 showing significant storage effects
- P_d = 3.2 is larger than 1/30 showing the dominance of inertia

Conclusions and Recommendations

- Wave speed and attenuation depend on four physical parameters or three dimensionless parameters χ, P_d, and F.
- Tidal waves and Hurricane Irma surge along the C-111 canal are decided primarily by inertia, gravity, and bank storage and less by canal bed friction.
- The analytical solutions for wave speed and decay can be used when testing numerical models for surge propagation.
- The analytical solutions can be used when determining aquifer and canal parameters of calibrating numerical models.

References

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