High-Resolution X-ray Computed Tomography of Macroporous Karst for Permeability Measurement and Non-Darcian Flow via Lattice Boltzmann Models

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Objectives

• Compute K of highly macroporous karst rock
• Compute K under different hydraulic gradients (different Re). Expect reduced apparent K at high Re due to eddy head dissipation
Introduction

• LBM is a mesoscopic method based on the scale between molecular dynamics and familiar continuum approaches

• A particle stream-and-collide perspective with interparticle forces is adequate for most simulations

• LBMs handle complex geometries well
Kinetic Theory

- Complete set of position ($x$) and momentum ($p$) coordinates for all particles gives dynamical state of system
- Together with classical mechanics, allows prediction of future states
- However, this level of description is not possible
- Use a statistical description: focus on the distribution function of the “state” of molecules

$$f(x, p, t)$$
LBM Basics

D2Q9

Discrete Velocities
Directional densities

Histogram view of the distribution function, \( f_a \).

Macroscopic density \( \rho = \sum_{a=0}^{8} f_a \)
Macroscopic velocity \( \mathbf{u} = \frac{1}{\rho} \sum_{a=0}^{8} f_a \mathbf{e}_a \)

D3Q19
Streaming \[ f_a(x + e_a \Delta t, t + \Delta t) = f_a(x, t) \]
Single Relaxation Time BGK (Bhatnagar-Gross-Krook) Approximation

\[ f_a(x + e_a \Delta t, t + 1) = f_a(x, t) - \frac{f_a(x, t) - f_a^{eq}(x, t)}{\tau} \]

Collision (i.e., relaxation towards equilibrium)

\[ f_a^{eq}(x) = w_a \rho(x) \left[ 1 + 3 \frac{e_a \cdot u}{c^2} + \frac{9}{2} \frac{(e_a \cdot u)^2}{c^4} - \frac{3}{2} \frac{u^2}{c^2} \right] \]

Streaming

- \(\tau\) relaxation time (viscosity and diffusion)
- \(c\) speed on lattice (1 lu /time step)

Collision and streaming steps must be separated if solid boundaries present (bounce back boundary is a separate collision)

- \(w_a\) are 4/9 for the rest particles (a = 0),
- 1/9 for \(a = 1, 2, 3, 4\), and
- 1/36 for \(a = 5, 6, 7, 8\).
Poiseuille flow in a circular pipe

\[ u_x = \frac{G}{4\mu} \left( a^2 - r^2 \right) \]
Poiseuille flow in a rectangular duct

\[ u_x(y, z) = -\frac{1}{2\mu} \frac{\partial p}{\partial x} a^2 \left[ 1 - \left( \frac{z}{a} \right)^2 + 4 \sum_{k=1}^{\infty} \frac{(-1)^k}{\alpha_k^3} \frac{\cosh\left( \frac{\alpha_k y}{a} \right)}{\cosh\left( \frac{\alpha_k b}{a} \right)} \cos\left( \frac{\alpha_k z}{a} \right) \right] \]

\[ \alpha_k = (2k - 1) \frac{\pi}{2}, \quad k = 1, 2, \ldots \]

For square, \( a = b \)

Why use LBM in macroporous karst context?

- Easy to incorporate complex geometry
- Transition to higher Reynolds numbers
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Re = 88
Data and Tools

• Karst data scales
  – 0.0003 to 0.3 m high-resolution CT scans
  – 0.002 to 30 m borehole imagery
  – 1 to 1000 m cave diver sonic rangefinder data

• Medium simulation required for borehole and rangefinder data

• LBM integrative tool
  – Compute K at multiple scales
  – Assess non-Darcy potential and impacts
Example Data Set

Burrow porosity in Miami Limestone barrier bar deposited during the last interglacial

(maximum unit thickness ~ 1m)

Photo: Mike Wacker/USGS
8- and 16-bit slices

- With 0.8 mm slice spacing, 401 slices = 321 mm

Data and image produced at the High-Resolution X-ray Computed Tomography Facility of the University of Texas at Austin
Thresholding ($<75/255 \rightarrow \text{pore}$)

Data and image produced at the High-Resolution X-ray Computed Tomography Facility of the University of Texas at Austin
Bulk of Sample and Experimental Cube

~22 million cells. Limit set by memory of computer and code.

Data produced at the High-Resolution X-ray Computed Tomography Facility of the University of Texas at Austin
Velocity Magnitude

Data produced at the High-Resolution X-ray Computed Tomography Facility of the University of Texas at Austin
Darcy’s Law

\[ q = -K \nabla h \]

\[ q = -k \frac{\rho g}{\mu} \nabla h \]

\[ q = -k \frac{1}{\mu} \nabla p \]

- \( h \) head \( = p/\rho g \)
- \( K \) hydraulic conductivity \( (LT^{-1}) \)
- \( q \) flux
- \( k \) permeability
- \( \rho \) density
- \( \mu \) viscosity
- \( p \) pressure
- \( g \) gravity
Hydraulic Conductivity Values

Freeze and Cherry (1979)
Groundwater, Prentice-Hall

Maximum k from air permeability laboratory measurements (Cunningham et al., 2006)
Darcy-Forschheimer Equation

• Darcy:

\[ \frac{\mu}{k} q = -\nabla p \]

• +Non-linear drag term:

\[ \frac{\mu}{k} q + a |q| q = -\nabla p \]
**Apparent K as a function of hydraulic gradient**

- Gradients could be higher locally
- Expect leveling at higher gradient?

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**Darcy-Forchheimer Equation**
Streamlines at different Reynolds Numbers

- Streamlines traced forward and backwards from eddy locations and hence begin and end at different locations

- $Re = 0.31$
  - $K = 34 \text{ m/s}$

- $Re = 152$
  - $K = 20 \text{ m/s}$
Conclusions

• LBM can measure permeabilities outside the range routinely accessible to laboratory measurements
• LBM can assess magnitude of departure from Darcy flow